MODELLING AND ROBUST PD COMPENSATION OF TWO-LINK FLEXIBLE MANIPUALTOR

A PROJECT THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF TECHNOLOGY

IN

ELECTRICAL ENGINEERING

BY

ANURAAG PARIDA

(107EE022)

SUBHAKANTA RANASINGH

(107EE053)

Department of Electrical Engineering

National Institute of Technology

Rourkela

2011

MODELING AND ROBUST PD COMPENSATION OF TWO-LINK FLEXIBLE MANIPUALTOR

A PROJECT THESIS SUBMITTED IN THE PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF TECHNOLOGY
IN
ELECTRICAL ENGINEERING

BY

ANURAAG PARIDA

(107EE022)

SUBHAKANTA RANASINGH

(107EE053)

UNDER THE GUIDANCE OF

PROF. BIDYADHAR SUBUDHI

PROF. ARUN GHOSH

Department of Electrical Engineering

National Institute of Technology

Rourkela -2011

NATIONAL INSTITUTE OF TECHNOLOGY

CERTIFICATE

This is to certify that the thesis entitled, "**Modeling and Robust PD Compensation of Two-Link Flexible Manipulator**" submitted by Anuraag Parida and Subhakanta Ranasingh in partial fulfillment of the requirements for the award of bachelor of technology degree in Electrical Engineering at the National Institute of Technology, Rourkela(Deemed University) is an authentic work carried out by them under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any degree or diploma.

Prof. Arun Ghosh
Department of Electrical Engineering
National Institute of Technology
Rourkela -769008

Prof. Bidyadhar Subudhi
Department of Electrical Engineering
National Institute of Technology
Rourkela -769008

Date:
Place:

Date:
Place:

ACKNOWLEDGEMENT

We are extremely fortunate to be involved in such an exciting and challenging research project like, "**Modeling and Robust PD Compensation of Two-Link Flexible Manipulator**". This project has increased our analyzing and understanding capability in the field of control system**.**

we would like to express our greatest gratitude and respect to our project guides Prof. Bidyadhar Subudhi and Prof. Arun Ghosh sir for their excellent guidance and valuable support. we consider ourselves extremely lucky to be able to work under their guidance .

we would also like to express our thanks to all our classmates, staffs and faculty members of Electrical Engineering department for making our stay in N.I.T. Rourkela a pleasant and memorable experience and also giving us an absolute working environment.

Anuraag Parida

Roll No: 107EE022

B.Tech , Electrical Engineering

Subhakanta Ranasingh

Roll No: 107EE053

B.Tech ,Electrical Engineering

Place:

Date:

Place:

Date:

ABSTRACT

The Two Link Flexible manipulator (TLFM) is a two-input-two-output, highly nonlinear and unstable system. Therefore, modeling and control system design of such a system is a challenging task. To this end, first, to establish a non-linear mathematical model of the TLFM, its kinematic and dynamic motions are analyzed through assumed mode method. Then a linearized model about equilibrium point at origin is obtained from this nonlinear model via "linmod/linearize" command in MATLAB. Next, for this linearized model a two-loop robust PD controller is designed via root locus based loop shaping approach. The controller obtained thus is employed for the nonlinear system and the robustness results are verified.

CONTENTS

List of Figures & Tables

CHAPTER - 1

INTRODUCTION

Introduction

Flexible robot manipulators, unlike industrial robots, are utilized for specific purposes like in a space shuttle arm. These flexible robots have an increased payload carrying capacity, lesser energy consumption, cheaper construction, faster movements, and longer reach. However, link flexibility causes significant technical problems. The weight reduction leads the manipulator to become more flexible and more difficult to control accurately. The manipulator being a distributed parameter system, it is highly non-linear in nature. Control algorithms will be required to compensate for both the vibrations and static deflections that result from the flexibility. This provides a challenge to design control techniques that gives precise control of desired parameters of the system in desired time, ability to cope up with sudden changes in the bounded system parameters, and robust performance.

In past few years modeling of two-link flexible manipulators has been carried out through different modeling approaches like assumed mode method (AMM) approach and finite element (FE) approach. In this thesis we adopted the AMM based modeling as it has many advantages over FE approach. One of the advantages is that AMM method describes the flexibility and vibration modes more descriptively than FE method. The nonlinear model is linearized about operating points to obtain linear model of the system.

Ultimate goal of such robotic designs is to accurate tip position control in spite of the flexibility in a reasonable amount of time. Conventional control system design is generally a trial and error process which is often not capable of controlling a process, which varies significantly during operation. Many controller algorithms such as adaptive control, Neural Network (NN), fuzzy logic have been used for tip position control of two-link flexible manipulators. These methods are effective but are more complex and difficult to design and analyze. So a simple PD controller is implemented on the dynamic model of the flexible manipulator.

The proportional–derivative (PD) controllers have been widely used in industry for many years due to their simplicity of operation, robustness of performance. Unfortunately, it has been a problem to achieve optimal PD gains because many industrial plants are often burdened with problems such as high order, time delays, and nonlinearities. For a wide range of practical applications, the tuning approaches like trial and error method and Ziegler-Nichols method works quite well. However, the tuning process is too laborious and time consuming. Due to these reasons, it is highly desirable to find new approaches to the tuning of PD controllers. In this thesis a new PD controller design is done via root locus based loop shaping approach.

CHAPTER - 2

MODEL DESCRIPTION

2.1 System Description

2.2 Mathematical Modeling

2.3 State-space Modeling

2.1 System Description

The Two Link Flexible Manipulator (2LFM) Robot is depicted in Fig.2.1. This robot system consists of two DC motors driving via harmonic gearboxes. Both links are flexible and instrumented with strain gauges. The primary link is rigidly clamped to the first drive (a.k.a. elbow) and carries at its end the second harmonic drive (a.k.a. shoulder) to which another flexible link is attached. Both motors are instrumented with quadrature optical encoders. Each flexible link is equipped with one strain gauge sensor which is located at the clamped end of the link. The described robotic mechanism emulates torsional compliance and serial linkage flexibility, which are common characteristics in mechanical systems such as robot arms. Also this system is similar in nature to the control problems encountered in large light space structures where the weight constraints result in flexible structures that must be controlled using feedback techniques.

Fig.2.1: Two-Link Flexible Manipulator

2.2 Mathematical Modeling

2.2.1. Kinematic modeling

Consider a planar 2-link flexible arm with rotary joints between the two flexible links whose first link is clamped at its base on the rotor of a motor and second link is loaded with a point mass at its tip as shown in Fig.2.2.

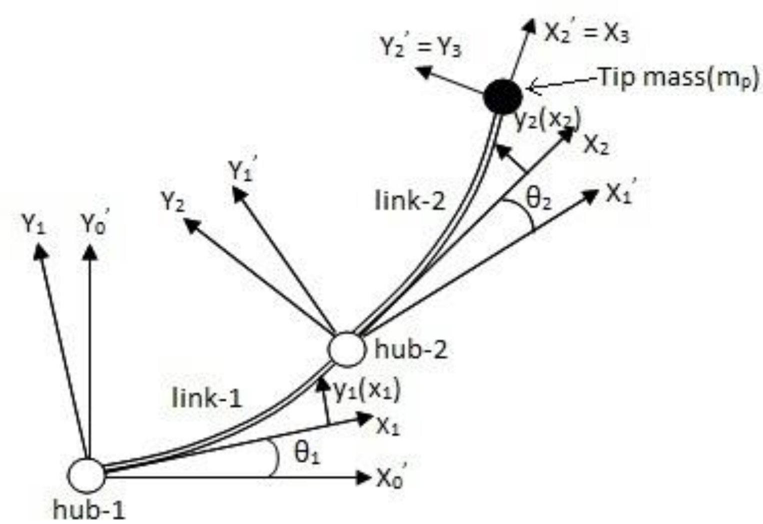


Fig.2.2: A Planar 2 Link Flexible Arm

The following coordinate frames are then established: the inertial frame (X_0', Y_0') ; the rigid body moving frame associated to link 1(X_1 ,Y_1) and link 2 (X_2 ,Y_2) ; and the flexible body moving frame associated to link 1(X_1', Y_1') and link 2 (X_2', Y_2'). The rigid motion is described by the joint angles $\theta_i$and $y_i(x_i)$ stand for the transversal deflection of link *i* at abscissa x_i, $0 \leq x_i \leq l_i$, l_i being the link length.

Let ${}^{i}p_i(x_i)$ be the position of a point along the deflected link i with respect to frame (X_i , Y_i) and is given by ${}^{i}p_i(x_i) = (x_i \;\; y_i(x_i))^T$. It is the absolute position of the same point in frame (X_0', Y_0'). Also, ${}^{i}r_{i+1} = {}^{i}p_i(l_i)$ indicates the position of the origin of frame (X_{i+1} , Y_{i+1}), with respect to frame (X_i , Y_i), and r_i its absolute position in frame (X_0', Y_0') .

The rigid joint rotation matrix A_i, and the rotation matrix E_i of the flexible link at the end point are given by [1],

$$A_i = \begin{bmatrix} Cos\,\theta i & -Sin\,\theta i \\ Sin\,\theta i & Cos\,\theta i \end{bmatrix}, \qquad E_i = \begin{bmatrix} 1 & -y'_{ie} \\ y'_{ie} & 1 \end{bmatrix} \tag{1}$$

where, $y'_{ie} = (\partial y_i/\partial x_i)\,|_{x_i = l_i}$,

Using Denavit Hartenberg representation the absolute position vectors are expressed as [1]

$$p_i = r_i + W_i\,{}^{i}p_i \;\; , \qquad r_{i+1} = r_i + W_i\,{}^{i}r_{i+1} \tag{2}$$

W_i in the above expression is the global transformation matrix from (X_0', Y_0') to (X_i, Y_i).

And absolute angular velocity of frame (X_i, Y_i) is [1]

$$\dot{\alpha}_i = \sum_{j=1}^{i} \dot{\theta}_j + \sum_{k=1}^{i-1} \dot{y}'_{ke} \tag{3}$$

Here the upper dot denotes the time derivative. And the linear absolute velocity of an arm point is given by [1],

$$p_i = \dot{r}_i + \dot{W}_i\,{}^{i}p_i + W_i\,{}^{i}\dot{p}_i \tag{4}$$

And ${}^{i}\dot{r}_{i+1} = {}^{i}\dot{p}_i(l_i)$.

As the links are assumed to be inextensible (i.e.$\dot{x}_i = 0$) so, ${}^{i}\dot{p}_i(x_i) = (0 \;\; \dot{y}_i(x_i))^T$.

2.2.2 Dynamic modeling

The dynamic equation of motion for two link flexible manipulator is derived through Lagrangian approach following reference [1]. By computing the kinetic energy 'T' and potential energy 'U' of the system ,the lagrangian equation is formed which is given by , L = T-U.

The total kinetic energy of the system is given by the sum of the following components:

$$\mathrm{T} = \sum_{i=1}^{2} \mathrm{T}_{hi} + \sum_{i=1}^{2} \mathrm{T}_{li} + \mathrm{T}_{p} \tag{5}$$

The kinetic of the rigid body located at hub *i* of mass m_{hi} and moment of inertia j_{hi} is

$$\mathrm{T}_{hi} = \frac{1}{2}\mathrm{m}_{hi}\dot{\mathrm{r}}_i^{\,T}\dot{\mathrm{r}}_i + \frac{1}{2}\mathrm{J}_{hi}\ \dot{\alpha}_i^{\,2} \tag{6}$$

with $\dot{\alpha}_i$ as in equation (3), the kinetic for link *i* of linear density ρ_i is

$$\mathrm{T}_{li} = \frac{1}{2}\int_0^{l_i} \rho_i(x_i)\dot{\mathrm{p}}_i^{\,T}(x_i)\dot{\mathrm{p}}_i(x_i)dx_i \tag{7}$$

The kinetic energy associated with the payload of mass m_p and moment of inertia J_p located at the end of second link is

$$\mathrm{T}_{p} = \frac{1}{2}\mathrm{m}_{p}\dot{\mathrm{r}}_3^{\,T}\dot{\mathrm{r}}_3 + \frac{1}{2}\mathrm{J}_{p}(\dot{\alpha}_2 + \dot{y}'_{2e})^{2} \tag{8}$$

Following identities are used to solve equation (6)-(8)

$$A_i^T A_i = E_i^T E_i = S^T S = I$$

$$A_i^T \dot{A}i = S\dot{\theta}_i\,, \qquad E_i^T \dot{E}i = (I\dot{y}_{ie} + S)\acute{\dot{y}}_{ie} \tag{9}$$

The potential energy (without considering gravity i.e. horizontal plane motion) is given by

$$\mathrm{U} = \sum_{i=1}^{2} \mathrm{U}_i = \sum_{i=1}^{2} \frac{1}{2}\int_0^{l_i}(EI)_i(x_i)\left[\frac{d^2 y_i(x_i)}{dx_i^{\,2}}\right]^2 dx_i \tag{10}$$

where U_i is the elastic energy stored in link *i* and $(EI)_i$ being its flexural rigidity.

While modeling link deflection has been considered by means of assumed mode shapes with proper boundary conditions satisfying partial differential equation

$$(EI)_i \frac{\partial^4 y_i(x_i,t)}{\partial {x_i}^4} + \rho_i \frac{\partial^2 y_i(x_i,t)}{\partial t^2} = 0 \,, \quad i = 1, 2 \tag{11}$$

Using the assumed modes method (Meirovitch, 1970; Meyer, 1971), a solution of the dynamic equation of motion of the manipulator can be obtained as a linear combination of the product of admissible functions $\phi_{ij}\,(x_i)$ and time-dependent generalized coordinates $\delta_{ij}\,(t)$

$$y_i(x_i,t) = \sum_{j=1}^{2} \phi_{ij}\,(x_i)\;\delta_{ij}\,(t) \tag{12}$$

and here we have assumed two mode shapes.

The dynamic model of the two-link flexible manipulator is obtained using the Lagrange-Euler equations,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = f_i \quad , \qquad i = 1, 2, 3,\ldots,6 \tag{13}$$

where $\{ f_i \}$ are the generalized forces performing work on $\{q_i\}$.

As a result of this procedure, the equations of motion for a planar 2-link flexible arm can be written in the familiar closed form,

$$\mathrm{M}(q)\ddot{q} + h(q,\dot{q}) + Kq + D\dot{q} = Qu \tag{14}$$

where $q = (\theta_1,\theta_2,\delta_{11},\delta_{12},\delta_{21},\delta_{22})^T$ and u is the 2-vector of joint (actuator) torques. M is the positive- definite symmetric inertia matrix, h is the vector of coriolis and centrifugal forces, K is the stiffness matrix, and Q is the input weighting matrix (i.e. in the form $[I_{2x2}\; O_{(6-2)x2}]$) due to the clamped link assumption. The combined effect of joint viscous friction and link structural damping can be added as $\dot{q}$, where D is a diagonal matrix.

2.3 State-space Modeling

A state space model is constructed using a set of system variables which define the status of a process at any instant in time. In general, system behavior changes with time, and the information about this evolution of system status usually resides in the rate-of-change variables within a system or in combinations of these variables and their derivatives. These status variables are known as the state variables of the system and the set of state variables which describe the behavior of a system is termed the system state.

The state space modeling is done to make the system analysis simpler and to find the measure of the state w.r.t. time by solving the system ordinary differential equations (ODEs). Dynamic model of 2 link flexible manipulator is of 2nd order, hence for designing state space model we have assumed state variables as,

$$x_1 = \theta_1,\ x_2 = \theta_2,\ x_3 = \delta_{11},\ x_4 = \delta_{12},\ x_5 = \delta_{21},\ x_6 = \delta_{22},$$

$$x_7 = \dot{\theta}_1,\ x_8 = \dot{\theta}_2,\ x_9 = \dot{\delta}_{11},\ x_{10} = \dot{\delta}_{12},\ x_{11} = \dot{\delta}_{21},\ x_{12} = \dot{\delta}_{22}$$

The state vector is taken as:

$$x = [\, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12} \,]^T$$

The state space form for non-linear system is represented as,

$$\dot{x} = f(x) + g(x).u$$

$$y = h(x, u) \tag{15}$$

For state-space representation the equation of motion is rewritten in the form shown below,

$$\ddot{q} = M(q)^{-1}(Qu - h(q,\dot{q}) - Kq - D\dot{q}) \tag{16}$$

Solving equation (16) and comparing with equation (15) , we get following results ,

$f(x) = [x_7 \;\; x_8 \;\; x_9 \;\; x_{10} \;\; x_{11} \;\; x_{12} \;\; a_1 \;\; a_2 \;\; a_3 \;\; a_4 \;\; a_5 \;\; a_6]^T$

$$g(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \\ b_{51} & b_{52} \\ b_{61} & b_{62} \end{bmatrix} \tag{17}$$

$u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$ (torques at two joints)

where, $a = [M(q)^{-1}(-h(q,\dot{q}) - Kq - D\dot{q})]_{6x1}$

a_1 , a_2 , a_3 , a_4 , a_5 , a_6 are the elements of a_{6x1} matrix.

$b = M(q)^{-1}Q$ with

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

b_{11}, b_{12}, b_{21}, b_{22}, b_{31}, b_{32}, b_{41}, b_{42}, b_{51}, b_{52}, b_{61}, b_{62} are the elements of b_{6x2} matrix.

CHAPTER - 3

LINEARIZATION

3.1 Linearization of Dynamic Model

3.2 Linear State-space Model

3.3 Transfer Function Model

3.1. Linearization of Dynamic Model

For a nonlinear system, the state space form is given as shown in equation (15). And for a autonomous system u = 0. So $\dot{x} = f(x)$. The model is linearized about the operating point which is found out by solving the following equations

$$\dot{x} = 0 \Rightarrow f(x) = 0 \tag{18}$$

Solving Eq.(18) the operating points are found out to be at x = [0 0 0 0 0 0 0 0 0 0 0 0]^T

Theoretically let the operating points be (x_0, u_0). So, for any small change in the model should behave linearly about these operating points. Now the autonomous system state-space model is given by:

$$\dot{x} = f(x,u) \tag{19}$$

About operating points, $x = x_0 + \delta x$ and $u = u_0 + \delta u$

Hence,

$$\dot{x} = f(x_0 + \delta x,\ u_0 + \delta u) \tag{20}$$

Applying taylor series expansion

$$\dot{x} = f(x_0,u_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0 u_0} \delta x + \left.\frac{\partial f}{\partial x}\right|_{x_0 u_0} \delta u + \text{higher order terms} \tag{21}$$

Neglecting the higher order terms from the Eq.(21),

$$f(x_0 + \delta x\,,\, u_0 + \delta u) - f(x_0,u_0) = \left.\frac{\partial f}{\partial x}\right|_{x_0 u_0} \delta x + \left.\frac{\partial f}{\partial x}\right|_{x_0 u_0} \delta u \tag{22}$$

Assuming the left hand side term as $\dot{\delta}x$ and δx as x and δu as u, the above equation becomes

$$\dot{x} = Ax + Bu \tag{23}$$

where, $A = \left.\frac{\partial f}{\partial x}\right|_{x_0 u_0}$ and $B = \left.\frac{\partial f}{\partial x}\right|_{x_0 u_0}$

3.2 Linear State-space Model

Using the formulae described in the previous section the linear state-space matrices A B C D are found out to be,

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 14.34 & 115.6 & 0.3701 & 7.703 & 0 & 0 & 1.50 & 3.23 & 0.008 & 0.02 \\ 0 & 0 & 2094 & -2.517e4 & -58.33 & -2742 & 0 & 0 & 219.50 & -703.7 & -1.34 & -8.66 \\ 0 & 0 & -1881 & 2.19e4 & 193.4 & 4026 & 0 & 0 & -197.20 & 612.40 & 4.46 & -12.74 \\ 0 & 0 & 1558 & -1.904e4 & -67.85 & -1412 & 0 & 0 & 163.30 & 532.30 & -1.56 & -4.46 \\ 0 & 0 & 9.378 & -46.26 & -275.4 & -947.8 & 0 & 0 & 0.98 & -1.29 & -6.36 & -2.99 \\ 0 & 0 & 3.664 & -18.07 & -17.79 & -10250 & 0 & 0 & 0.38 & -0.50 & -0.41 & -32.44 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 9.78 & -4.485 \\ -4.485 & 2629 \\ -15.76 & -2302 \\ -9.034 & 1968 \\ -0.01973 & 3.109 \\ -0.007708 & 2.744 \end{bmatrix}$$

$$C=\begin{bmatrix} 0.5 & 0 & 0.186 & 0.215 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0.883 & -0.069 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{24}$$

The tip positions of both the links of the two-link flexible manipulator are taken as output.

3.3 Transfer Function Model

A Transfer function model is framed using previously obtained A, B, C, D matrices and considering the two torques required for link movement as two inputs and the measured tip positions of both the links as two outputs. So the two-link manipulator system is a two-input-two-output system. A transfer function matrix G(s) is computed which consists of four transfer functions defined as below:

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \tag{25}$$

where, $G_{11}(s)$ = (tip position of link-1) / (input torque-1),

$G_{12}(s)$ = (tip position of link-1) / (input torque-2),

$G_{21}(s)$ = (tip position of link-2) / (input torque-1),

and $G_{22}(s)$ = (tip position of link-2) / (input torque-2),

From this model, poles and zeroes are found out. The values of pole and zeroes which are approximately equal are then cancelled out. Finally a reduced order transfer function model is obtained.

Poles of the system:

0 , 0 , 0 , 0 , -692.6 , -16.26±99.93j , -28.66 , -3.31±16.23j , -3.95±8.18j

- *Zeroes of G_{11}:*

  Gain = 0.01641

  -39.2 , -9.525 , -2.88±15.52j , 15.89±98.93j , -437.05±431.96j , 0 , 0

- *Zeroes of G_{12}:*

  Gain = -7.356

  -2.294±6.04j , -90.15±9.433j , -3.45±16.59j , -15.95±99.93j , 0 , 0

- *Zeroes of G_{21}:*

  Gain = -2.259

  -184.6 , 76.55 , 19.17 , -4.846 , -1.136±15.566j , -16.3±99.97j , 0 , 0

- *Zeroes of G_{22}:*

  Gain = 1317

  -1.988±5.74j , -4.634±14.48j , -0.9365±15.2j , -16.26±99.93j , 0 , 0

The reduced order transfer function model obtained after pole zero cancellation is as follows:

$$\left.\begin{aligned}
G_{11} &= \frac{0.01641\,(S+39.2)(S+9.525)(S+437.05\pm431.96j)}{S^2(S+28.66)(S+692.6)(S+3.95\pm8.18j)} \\[2ex]
G_{12} &= \frac{-7.356\,(S+2.294\pm6.04j)(S+90.15\pm9.433j)}{S^2(S+28.66)(S+692.6)(S+3.95\pm8.18j)} \\[2ex]
G_{21} &= \frac{-2.259(S+184.6)(S-76.55)(S-19.17)(S+4.846)(S+1.136\pm15.566j)}{S^2(S+28.66)(S+692.6)(S+3.95\pm8.18j)(S+3.31\pm16.23j)} \\[2ex]
G_{22} &= \frac{1317\,(S+1.988\pm5.74j)(S+4.63\pm14.48j)(S+0.9365\pm15.2j)}{S^2(S+28.66)(S+692.6)(S+3.95\pm8.18j)(S+3.31\pm16.23j)}
\end{aligned}\right\} \tag{26}$$

CHAPTER - 4

CONTROLLER DESIGN

4.1 Controller Design

SISO controller design has been carried out for this system as standard methods are available and it is easy to design. The two-link flexible manipulator is a MIMO system as two input torques are required for both the links to rotate and the angles rotated by both links are sensed as the output. For controller design purpose the MIMO system is required to be converted into SISO system.

Direct two loop SISO controller design can be carried out for a two-input-two-output system only if the diagonal elements of the system matrix contains all the unstable poles and the also the dominant stable pole (nearer to imaginary axis in s-plane). It is observed that the reduced order two-link flexible manipulator model satisfies the above requirements of SISO controller design. So SISO controller can be used in the reduced order linear model of two-link flexible manipulator and for better output control it should be verified whether it yields good robustness.

Initially we are interested in two loop PID controller design, since PID controllers are well accepted in industries and other practical applications. But the reduced order transfer function model described in the previous section contains two poles at the origin. This indicates that the integral action of the PID controller is not required for two-link flexible manipulator system. So a simple PD controller is proposed for the tip position control of the two-link flexible manipulator. The transfer function of the PD controller is defined as;

$$C(s) = K_{PD}(s) = K_P + K_D.s \qquad (27)$$

where K_P is the proportional gain and K_D is the derivative gain. The proportional gain in the PD controller helps in reducing rise time. It also decreases the steady state error of the system. Derivative gain is necessary for increasing stability of the system and reducing the overshoot, and improving the transient response. Positive values of K_p and K_d are chosen to ensure the stability of the closed loop system.

The two-link flexible manipulator system requires two controlled input torques to be applied to both the links for tip position control. So two PD controllers have been designed for the system based on root locus loop shaping technique. This technique involves analysis of pole zero diagram and placing open loop poles in proper places in the s-plane so that the open loop transfer function satisfies certain sensitivity and robustness criteria.

The closed loop control of the two-link flexible manipulator consists of a negative feedback with loop gain 1 and feedforward PD controller alongwith the plant transfer function. The reduced order transfer function of the manipulator model is given by G(s) as obtain from the previous section and the controller transfer function is given by C(s), (27). So the open loop transfer function L(s) is defined as;

$$L(s) = G(s).C(s) \tag{28}$$

In loop shaping technique the main work is focused on the open loop transfer function L(s). Since G(s) of the manipulator model is known and fixed, the controller transfer function C(s) is to be chosen such that it shapes the loop function response L(s) in frequency domain for better sensitivity and robustness, i.e. the PD controller gains should be chosen properly so that the open loop poles will be placed at required places thus ensuring robustness to the system. Sensitivity and robustness analysis is explained in detail in the next section.

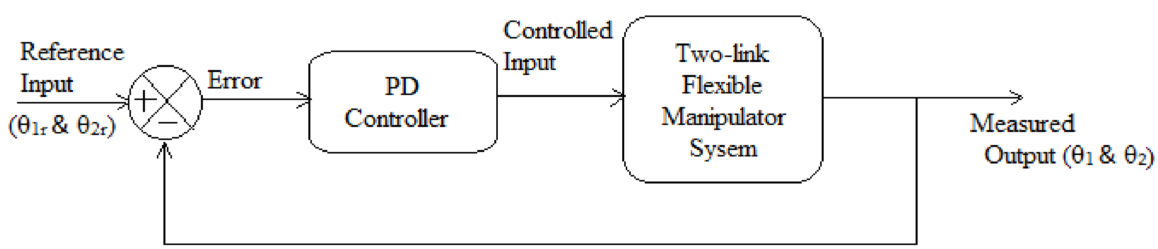


Fig.4.1. PD Controller design

4.2 Robustness Analysis

Robust stability, with respect to plant parameter variations, can be analyzed by the concept of sensitivity. Sensitivity is defined as the percentage change in loop transfer function of the plant caused by a differential change in any plant parameter. For any transfer function T(s, α) with α being the variable plant parameter, sensitivity is defined by the relationship:

$$S_\alpha^T \stackrel{\text{def}}{=} \lim_{\Delta\alpha\to 0} \frac{\Delta T/T}{\Delta\alpha/\alpha} = \frac{dT/T}{d\alpha/\alpha} = \frac{\alpha}{T}\frac{dT}{d\alpha} \tag{29}$$

Sensitivity function S(s) in terms of open loop transfer function is defined by:

$$S(s) = \frac{1}{1+L(s)} = \frac{1}{1+G(s)C(s)} \tag{30}$$

The sensitivity function S(s) quantifies the effect of the loop compensator function C(s) relative to unknown plant parameter variations.

The complementary sensitivity function T(s) is defined by:

$$T(s) = \frac{L(s)}{1+L(s)} = \frac{G(s)C(s)}{1+G(s)C(s)} \tag{31}$$

and it is also the closed loop transfer function from reference input to the output.

The maximum amplitude of S(s = jω) for all ω ≥ 0; i.e. $\max_\omega |S(j\omega)|$ is used to obtain simple bounds on both the gain and the phase margins of stable, closed-loop systems. Indeed, a single bound on $\max_\omega |S(j\omega)|$ can be used as a measure of robust stability in virtually all closed-loop stable cases, including the nonminimum phase and open-loop unstable cases in which both the GM and the PM in the bode plot are ill-defined.

For robustness analysis let's begin with the definition of the so-called "infinite-norm" of any stable, proper, rational transfer function T(s). The infinite-norm of a rational function T(s), which is both analytic and bounded in the half-plane Re(s) > 0, is defined by the relationship: [2]

$$\| T \|_\infty \overset{\text{def}}{=} \max_\omega |T(j\omega)| \tag{32}$$

and simply represents the maximum amplitude attained by its frequency response.

It is observed that the frequency response magnitude of the return difference, $| 1 + L(j\omega) | = | 1 + G(j\omega)C(j\omega) |$, equals the length of a vector drawn from -1 to L(jω)-plane; that is $|1+L(j\omega)| = |S(j\omega)|^{-1}$ represents the distance from the loop gain L(jω) to the critical -1 point in the complex L(jω)-plane for all $\omega \geq 0$. Therefore, the inverse of the infinite-norm of the sensitivity function, namely, $\|S\|_\infty^{-1}$ represents the minimum distance from L(jω) to the -1 point.

Since

$$\| S \|_\infty = \max_\omega |S(j\omega)| \overset{\text{def}}{=} \bar{S} \geq 1 \tag{33}$$

It follows that

$$\| S \|_\infty^{-1} = \bar{S}^{-1} = \min_\omega | 1+ L(j\omega) | \leq 1 \tag{34}$$

Therefore, a polar plot of L(jω) will just contact, but fail to penetrate, a circle of radius $\bar{S}^{-1}$, centered at -1 in the complex L(jω)-plane as shown in Fig.4.2.

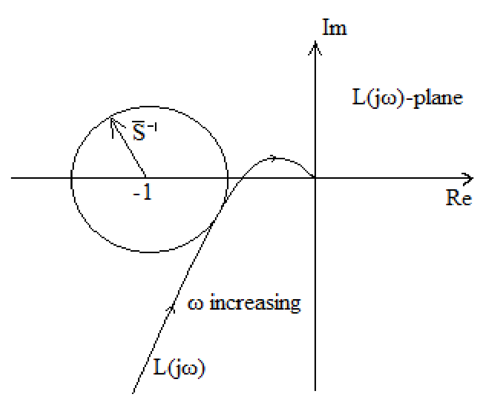


Fig.4.2: The Loop Gain and the $\bar{S}^{-1}$ circle[2]

Minimum bounds on both the gain margin and the phase margin of a system characterized by a minimum phase L(s) can be expressed directly as functions of $\bar{S}$, in light of Fig.4.3, where Fig.4.2 has been embellished with an arc of radius 1,centered at the origin, to define the angle θ. Therefore, the distance $l = 1 - \bar{S}^{-1}$ depicted in Fig.4.3 represents a lower bound on the gain margin of the system, in the sense that

$$\mathrm{GM} \geq \frac{1}{l} = \frac{\bar{S}}{\bar{S}-1} \tag{35}$$

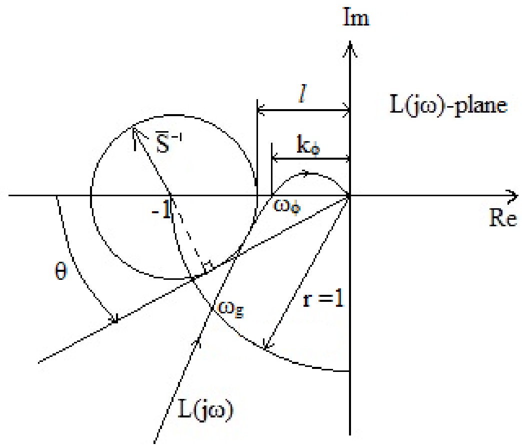


Fig.4.3: GM and PM from the $\bar{S}^{-1}$ circle [2]

Furthermore, the angle θ depicted in Fig.4.3 represents a lower bound on the phase margin of the system, in the sense that

$$\mathrm{PM} \geq \theta = 2\sin^{-1}\left(\frac{1}{2\bar{S}}\right) \tag{36}$$

To ensure an accepted nominal design that attains a GM ≥ 2 and a PM ≥ 30^0, as noted it may be required that

$$\bar{S} \leq 2 \approx 6 \text{ dB or that } \bar{S}^{-1} = \min_\omega |\, 1 + L(j\omega) \,| \geq 0.5 \tag{37}$$

so that the GM ≥ 2 and the PM ≥ $29^0 \approx 30^0$, in views of Eqs.(35) and (36), respectively. However, such a requirement may be conservative, since larger values of $\bar{S}$ can imply both a GM > 2 and a PM > 30^0. In particular, Eq.(37) ensures that L(jω) remains an acceptable, “marginal” distance away from the critical -1 point, irrespective of the number and the direction of encirclements required for closed-loop stability; i.e. Eq.(37) ensures robust stability w.r.t. plant parameter variations once nominal closed-loop stability is obtained.

So from the above analysis it is clear that the controller should be designed so that it ensures $\| S \|_\infty$ & $\| T \|_\infty < 2$. To take care of both the norms normally mixed sensitivity norm is used which is given by:

$$\Delta = \left\| \begin{matrix} S \\ T \end{matrix} \right\|_\infty \tag{38}$$

Therefore our requirement is to design a controller which yields $\| \Delta \|_\infty < 2$ for robust stability and performance.

For our system, plant transfer function is G(s) as obtained in chapter 3.3. So, G(s) is given by;

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

and two loop controller transfer function is given by;

$$C(s) = \begin{bmatrix} C_1(s) & 0 \\ 0 & C_2(s) \end{bmatrix}$$

where, $C_1(s) = K_{P1} + K_{D1}.s$ and $C_2(s) = K_{P2} + K_{D2}.s$

So, the open loop transfer function L(s) = G(s).C(s)

Sensitivity for our system is given by:

$$S(s) = (I + L)^{-1}$$

So the complementary transfer function T(s) = (I – S)

where I is the identity matrix of order 2.

Using "sigma[S:T]" command in MATLAB, the mixed sensitivity norm $\| \Delta \|_\infty$ for the TLFM system is plotted. Also the root locus plot for $G_{11}(s).C_1(s)$ and $G_{22}(s).C_2(s)$ is obtained using “rlocus” command and both the results are analyzed to get robust two loop PD gains. The results thus obtained are shown below:

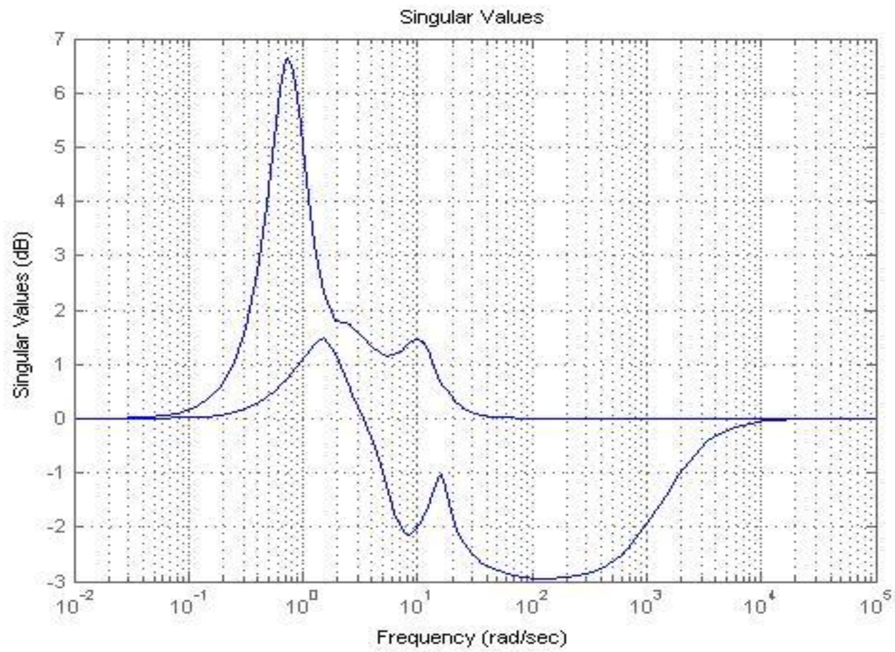


Fig.4.4: Mixed Sensitivity (Δ) plot for $K_{P1} = K_{P2} = 0.7$ and $K_{D1} = K_{D2} = 0.7$

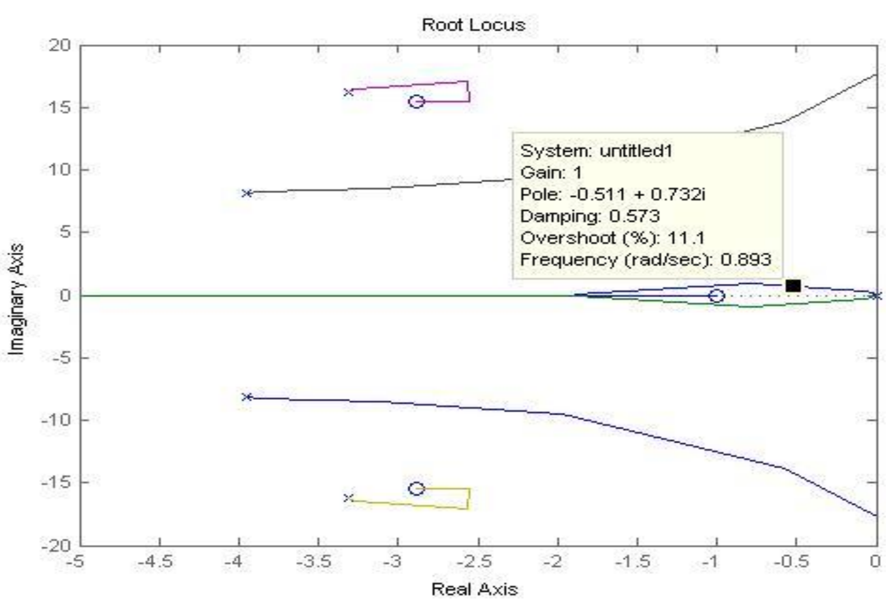


Fig.4.5: Root locus plot with $K_{P1} = K_{P2} = 0.7$ and $K_{D1} = K_{D2} = 0.7$

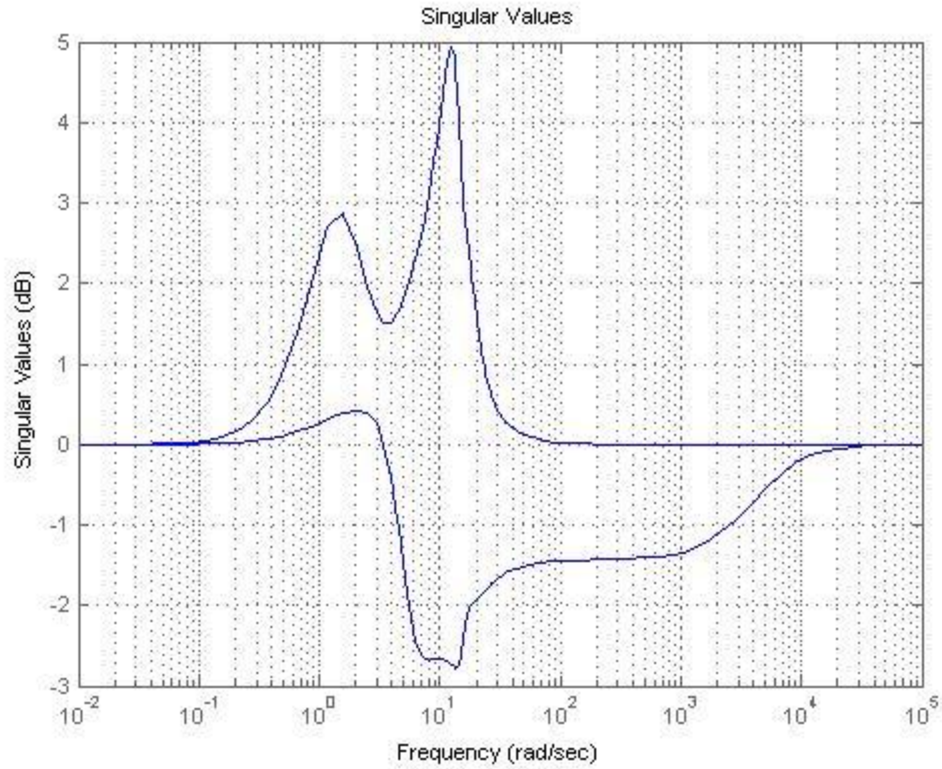


Fig.4.6: Mixed Sensitivity (Δ) plot for $K_{P1} = K_{P2} = 2.7$ and $K_{D1} = K_{D2} = 2.7$

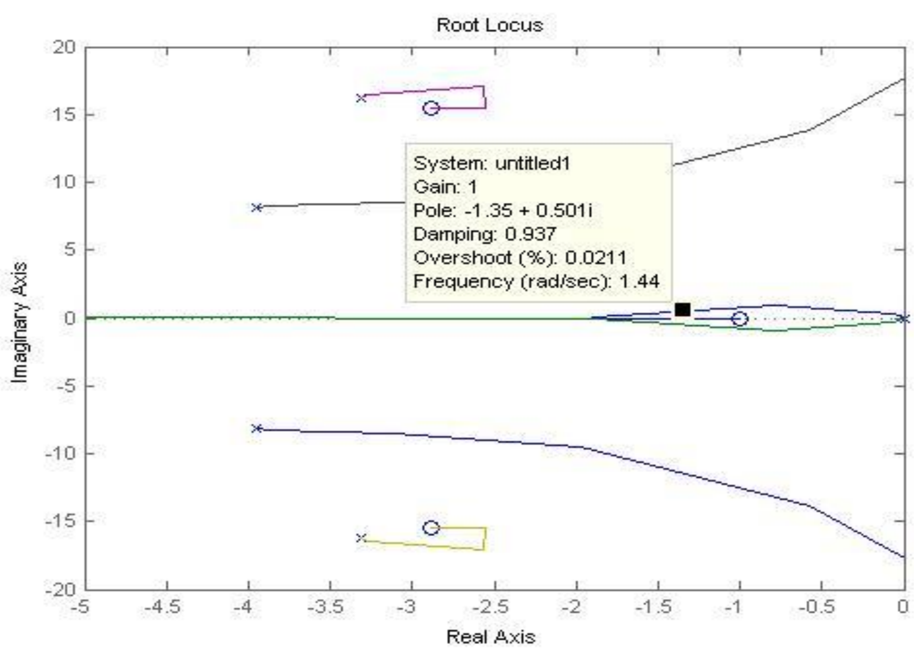


Fig.4.7: Root locus plot with $K_{P1} = K_{P2} = 2.7$ and $K_{D1} = K_{D2} = 2.7$

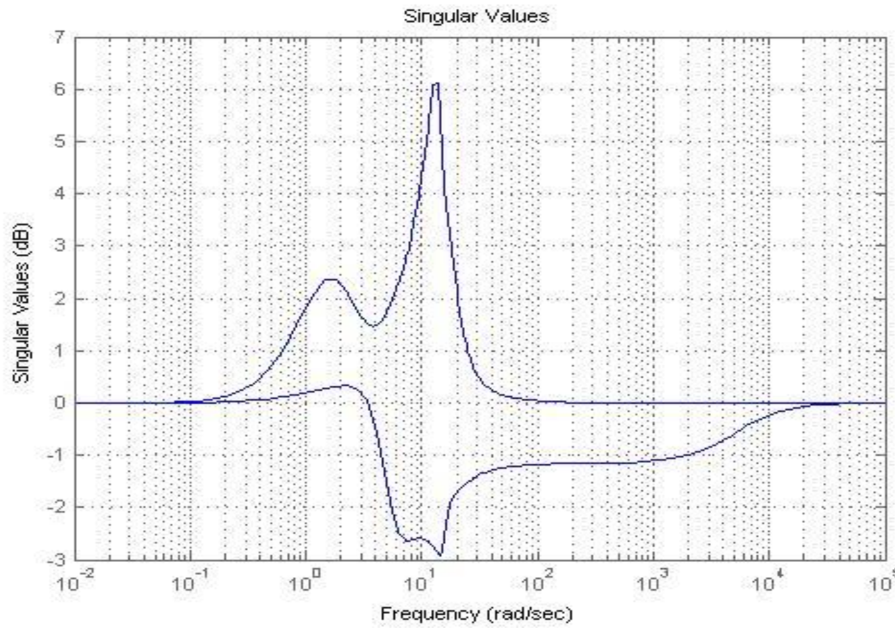


Fig.4.8: Mixed Sensitivity (Δ) plot for $K_{P1} = K_{P2} = 3.5$ and $K_{D1} = K_{D2} = 3.5$

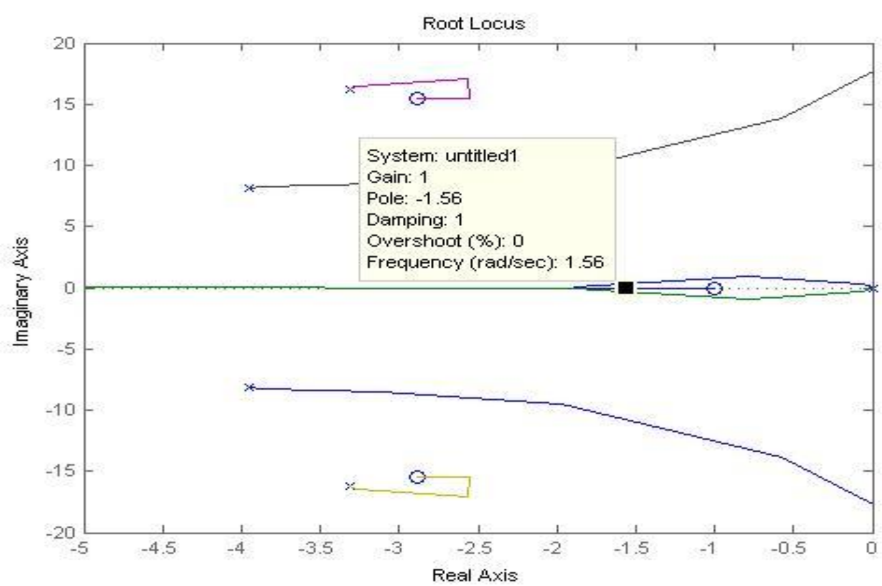


Fig.4.9: Root locus plot with $K_{P1} = K_{P2} = 3.5$ and $K_{D1} = K_{D2} = 3.5$

From the above mixed sensitivity and root locus plots it is observed that for PD gains [2.7, 2.7] the system exhibits good robust stability and less settling time and for higher PD gains mixed sensitivity margin crosses 6dB line making the system robustly unstable.

CHAPTER - 5

SIMULATION RESULTS

5.1 Open Loop Responses of Nonlinear Model

5.2 Open Loop Responses of Linear Model

5.3 Closed Loop Responses of PD Compensated Model

5.1 Open Loop Responses of Nonlinear Model

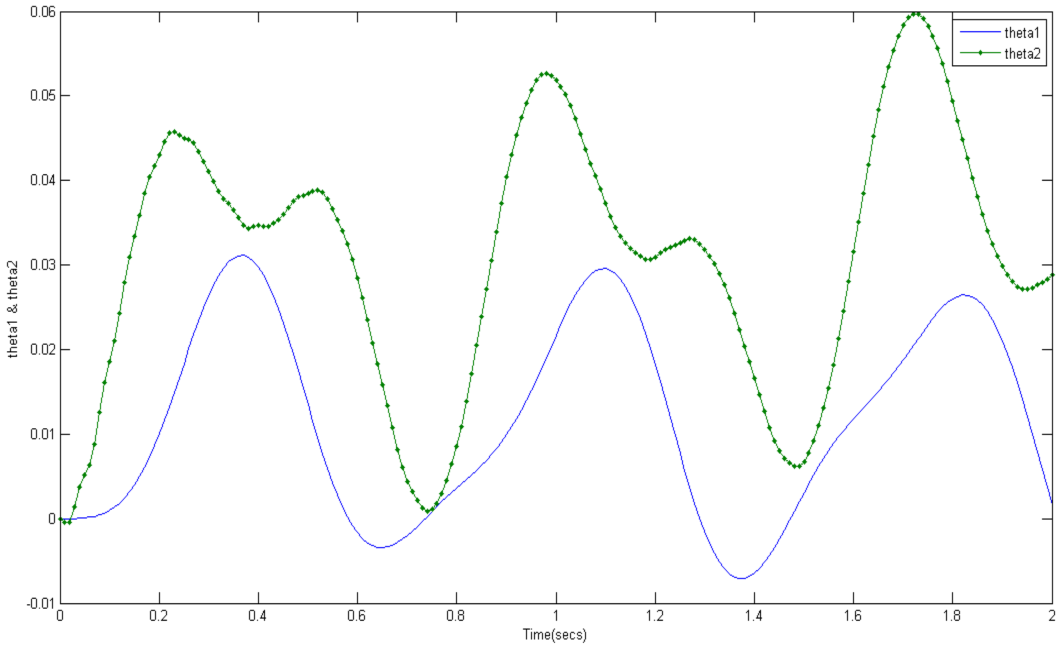
In this chapter simulation of the dynamic model output responses are presented. The nonlinear state-space model of the two-link flexible manipulator dynamic equation described in chapter 2 is simulated using 'ODE45' command in MATLAB. The system parameters considered for simulation is given in the table below.

TABLE 1

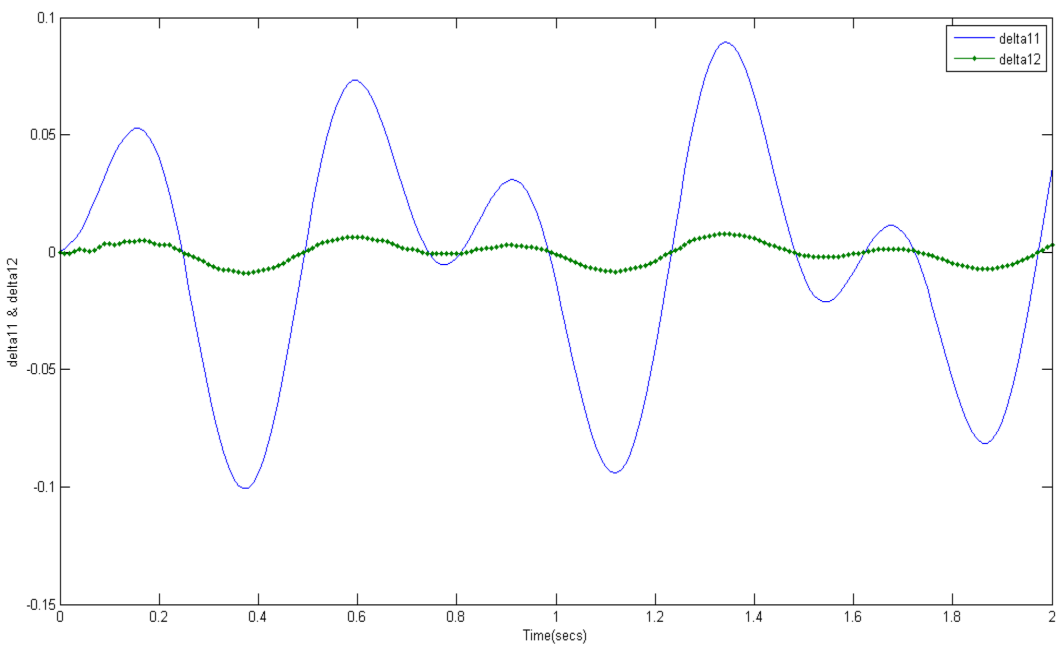
SYSTEM PARAMETERS

$\rho 1$	0.2 kg/m	$\rho 2$	0.2 kg/m
L1	0.5 m	L2	0.5 m
M1	0.1 kg	M2	0.1 kg
J_{o1}	0.0083 kg m^2	J_{o2}	0.0083 kg m^2
J_{h1}	0.1 kg m^2	J_{h2}	0.1 kg m^2
M_{h2}	1 kg	Mp	0.1 kg
$EI_1 = EI_2$	1 N m^2	Jp	0.0005 kg m^2

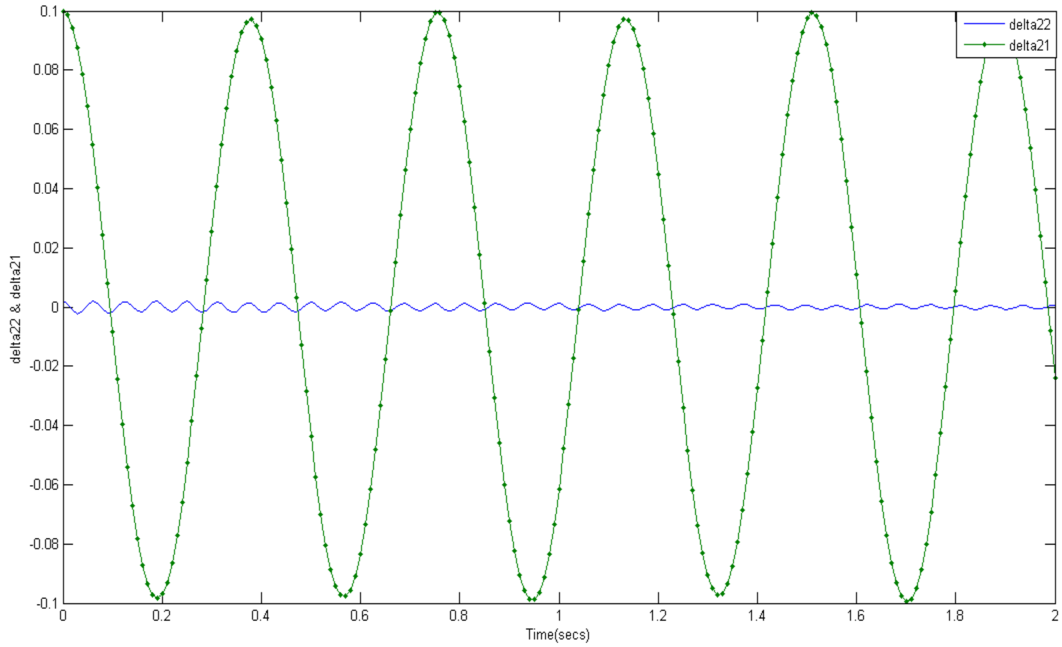
Simulating the nonlinear state space model for 2 seconds using different initial conditions in MATLAB following results were obtained,


theta1
theta2
theta1 & theta2
Time(secs)
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

(a)


delta11
delta12
delta11 & delta12
Time(secs)
0.1
0.05
0
-0.05
-0.1
-0.15
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

(b)

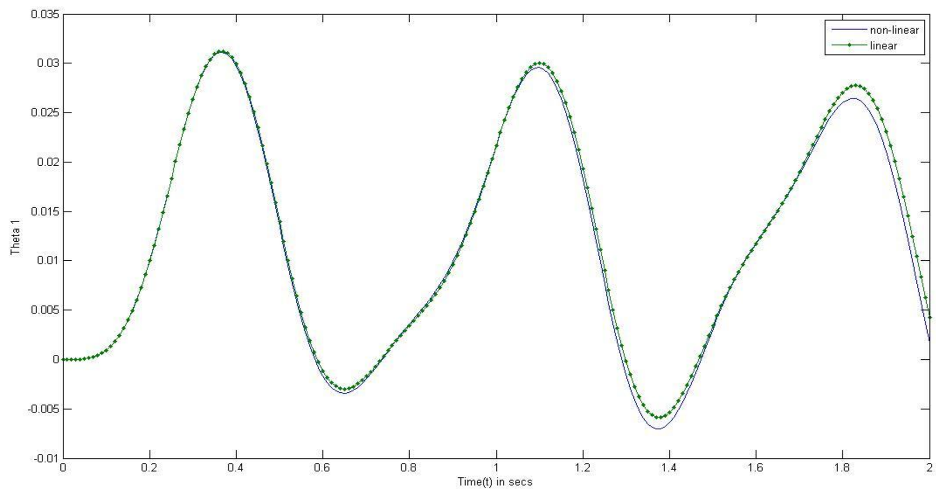


(c)

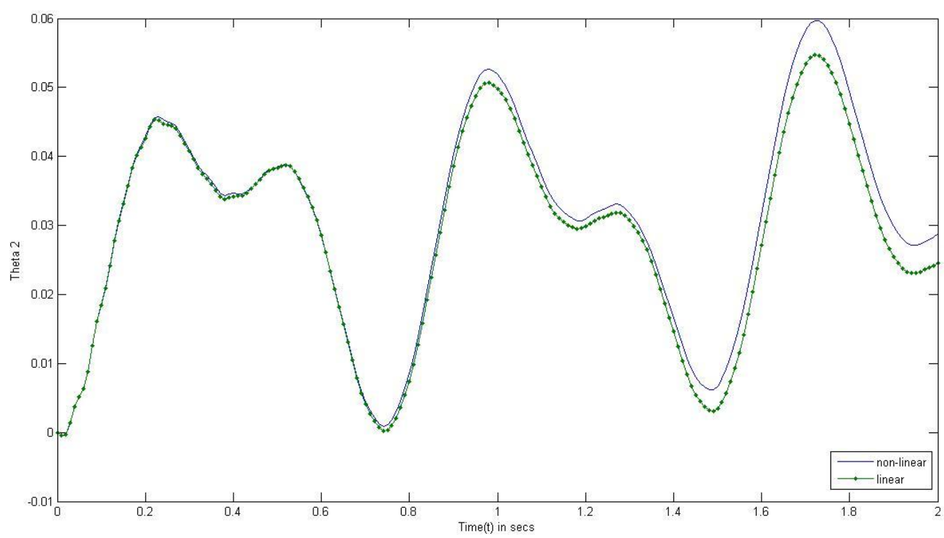
Fig.5.1: (a) Joint evolution ($\theta_2(0) = 0$, $\delta_{21}(0) = 0.1$, $\delta_{22}(0) = 0.002$) ,(b) Deflections of link 1 ($\theta_2(0) = 0$, $\delta_{21}(0) = 0.1$, $\delta_{22}(0) = 0.002$), (c) Deflections of link 2 ($\theta_2(0) = 0$, $\delta_{21}(0) = 0.1$, $\delta_{22}(0) = 0.002$)

5.2 Open Loop Responses of Linear Model

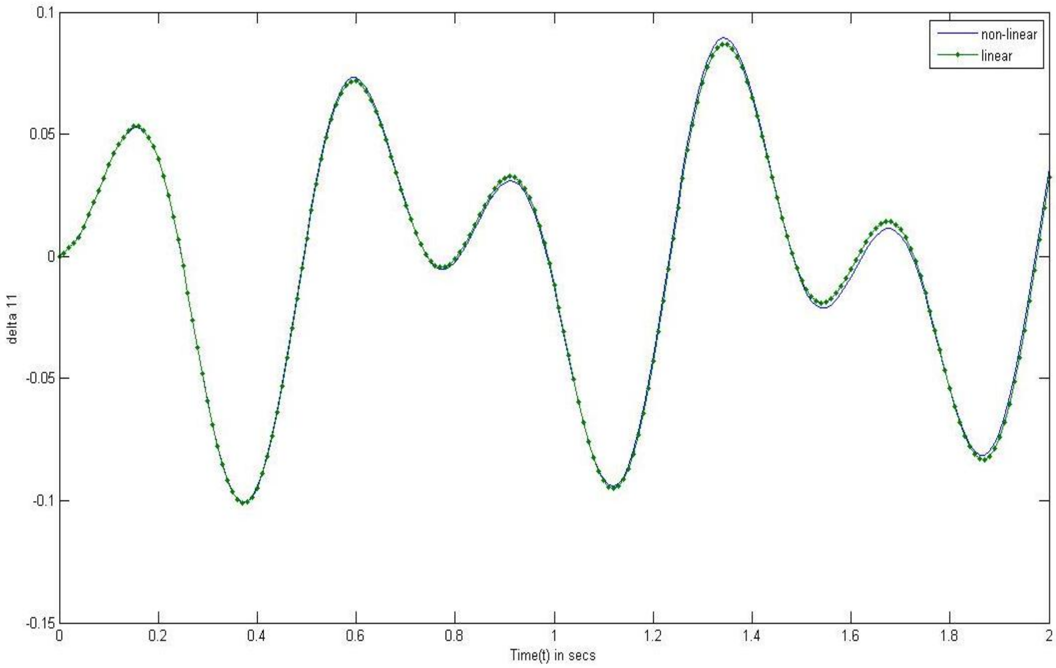
Now the output responses of linear state-space model obtained in chapter 3.2 is superimposed with that of the nonlinear state-space model with initial conditions as $\delta_{21}(0) = 0.1$ and $\delta_{22}(0) = 0.002$ and simulation time 2 sec at 10 msec integration step;



(a)

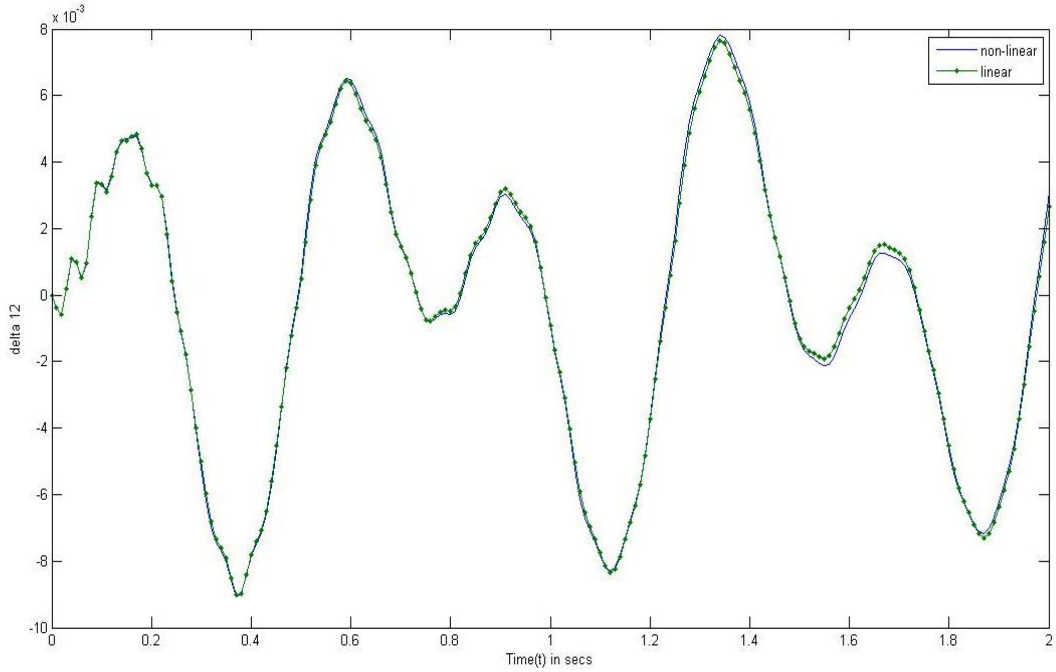


(b)



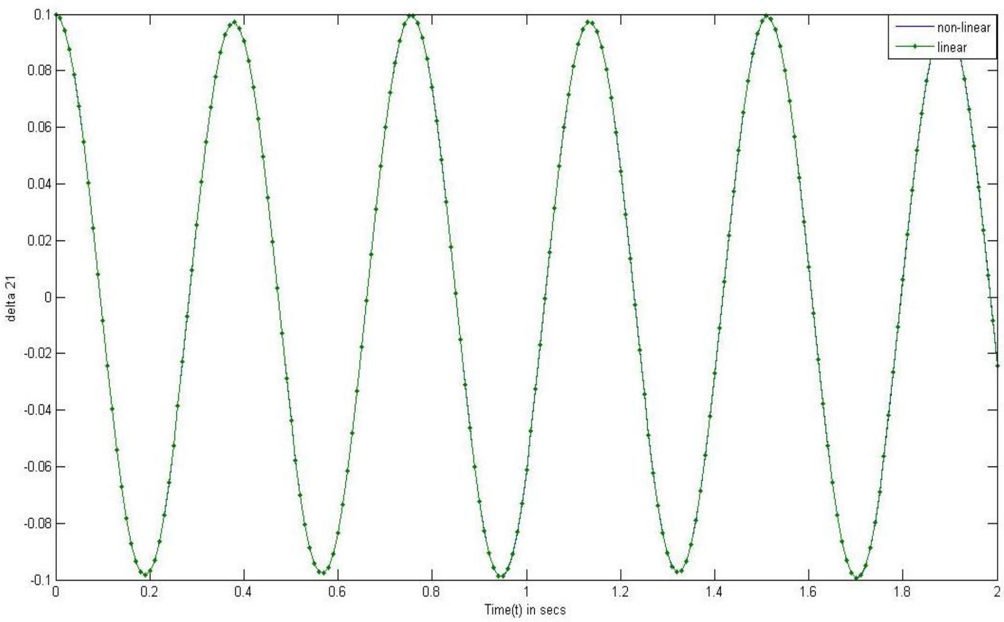
non-linear
linear
delta 11
Time(t) in secs
0.1
0.05
0
-0.05
-0.1
-0.15
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

(c)

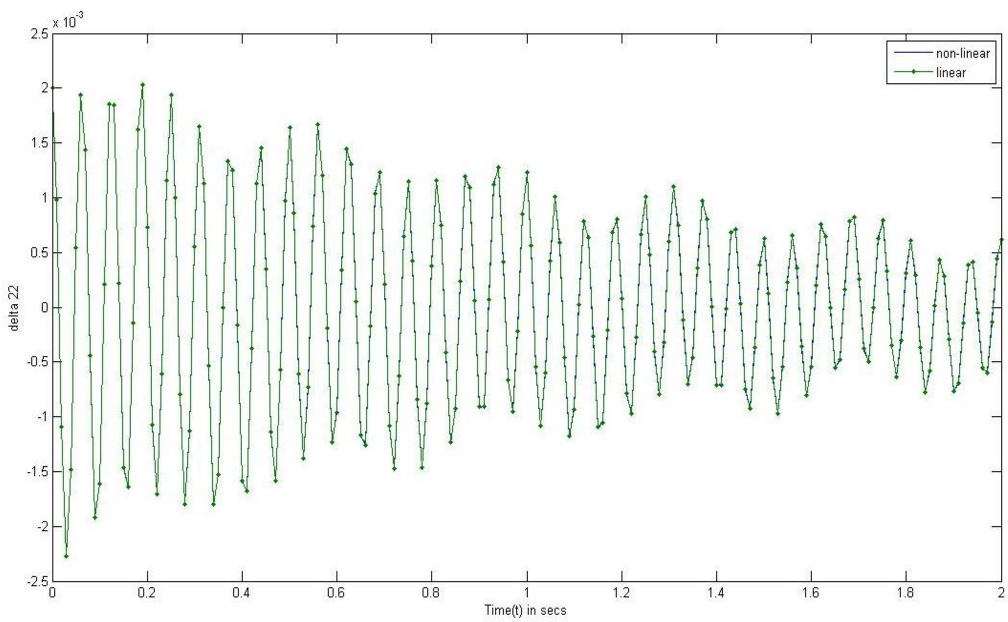


x 10^-3
non-linear
linear
delta 12
Time(t) in secs
8
6
4
2
0
-2
-4
-6
-8
-10
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

(d)



(e)



(f)

Fig.5.2: (a) Link-1 Joint evolution, (b) Link-2 Joint evolution (c), (d) Deflections of Link-1, (e), (f) Deflections of Link-2.

5.3 Closed Loop Responses of PD Compensated Model

A simulink model is made using MATLAB having three different models; nonlinear state-space model, transfer function model & linear state-space model and two loop PD controllers for each of the models. The simulink block diagram is shown below;

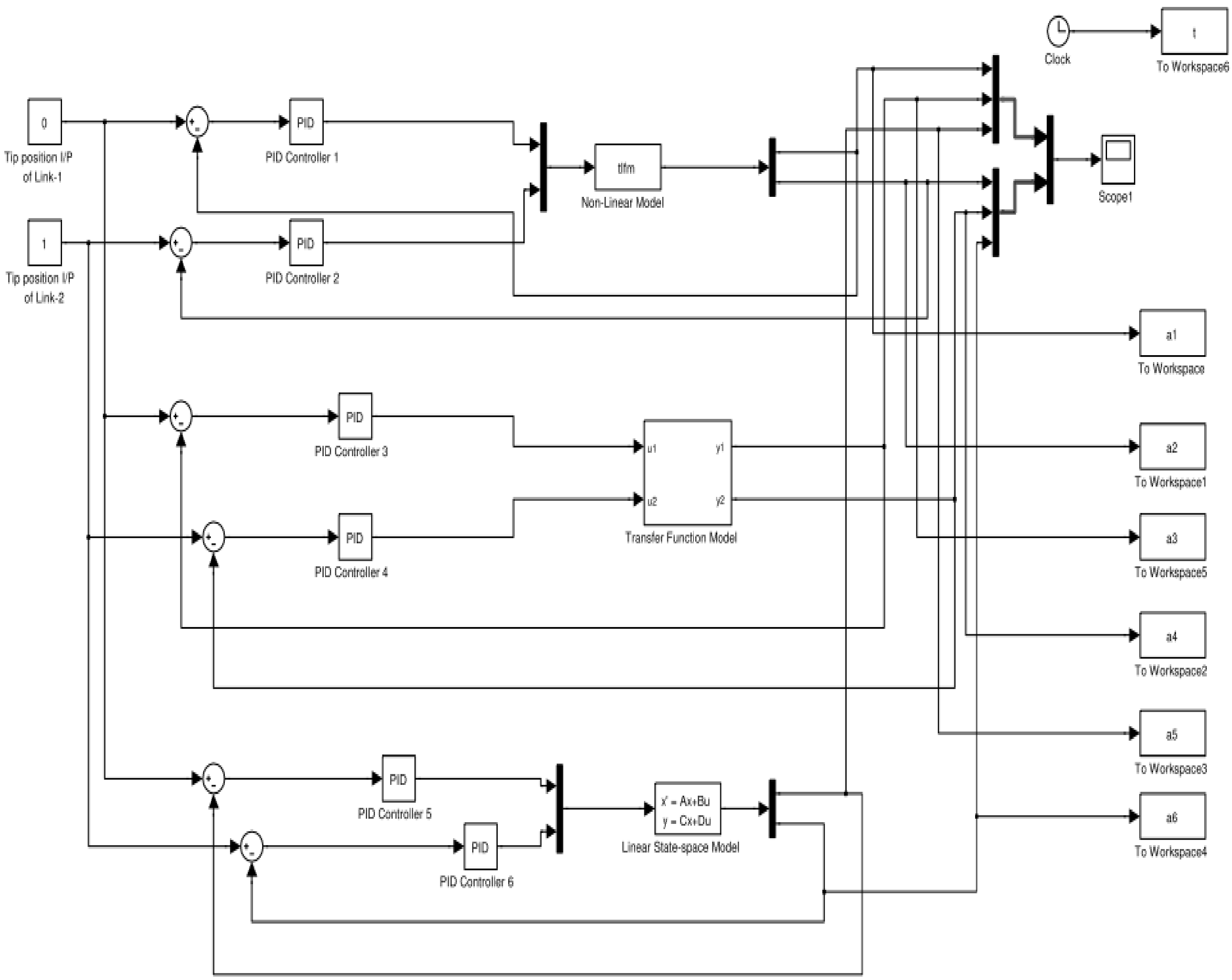


Fig.5.3: Simulink Block Diagram

The tip positions of both the links of the two-link flexible manipulator for PD controller gains obtained from robust analysis ($K_p = 2.7$ and $K_d = 2.7$) are plotted in the following figures,

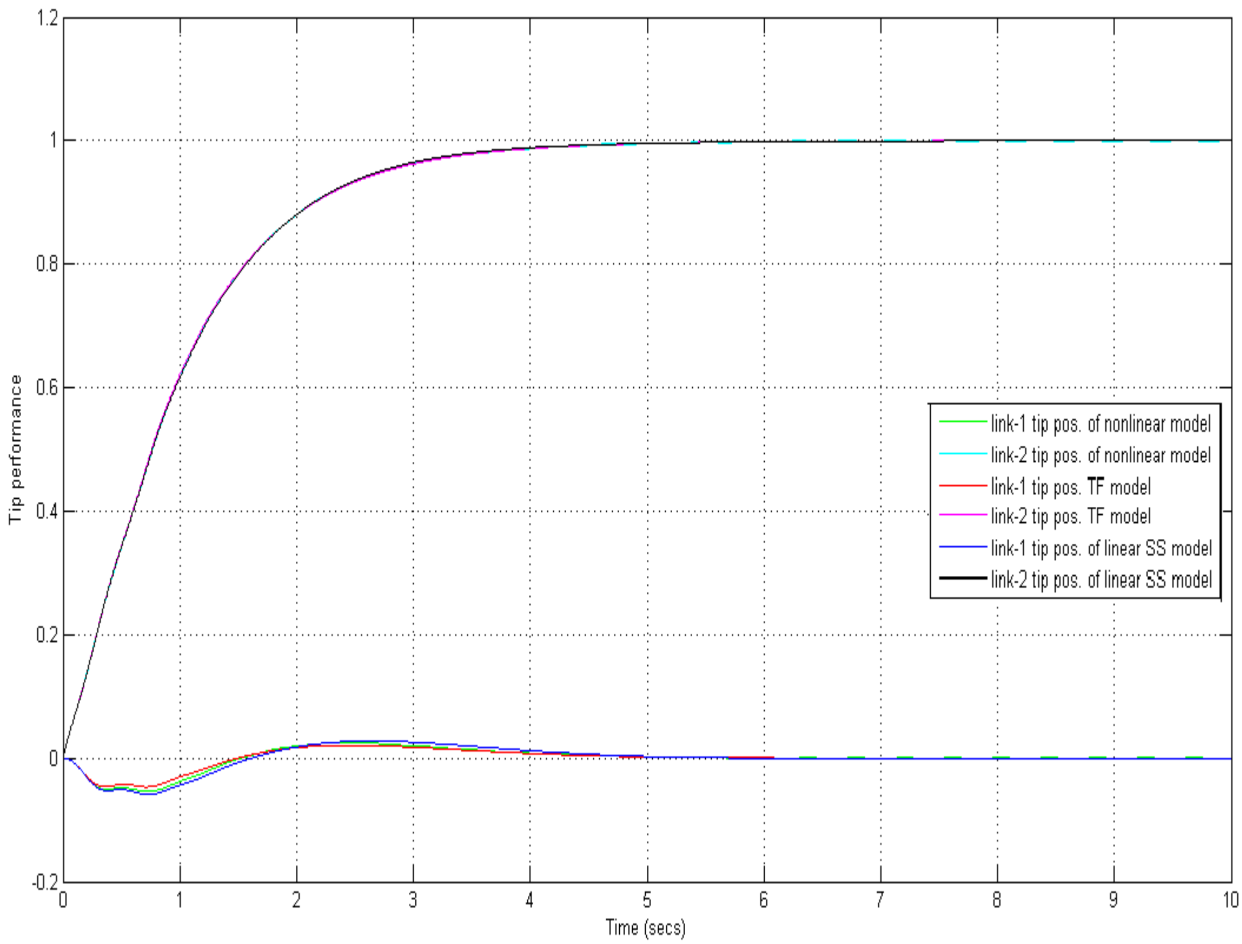


Fig.5.4: Tip performances of 3 different models using PD controller gains $K_p = 2.7$ and $K_d = 2.7$ with reference inputs to link-1 tip position as 0 and link-2 tip position as 1 for $M_P = 0.1$ kg.

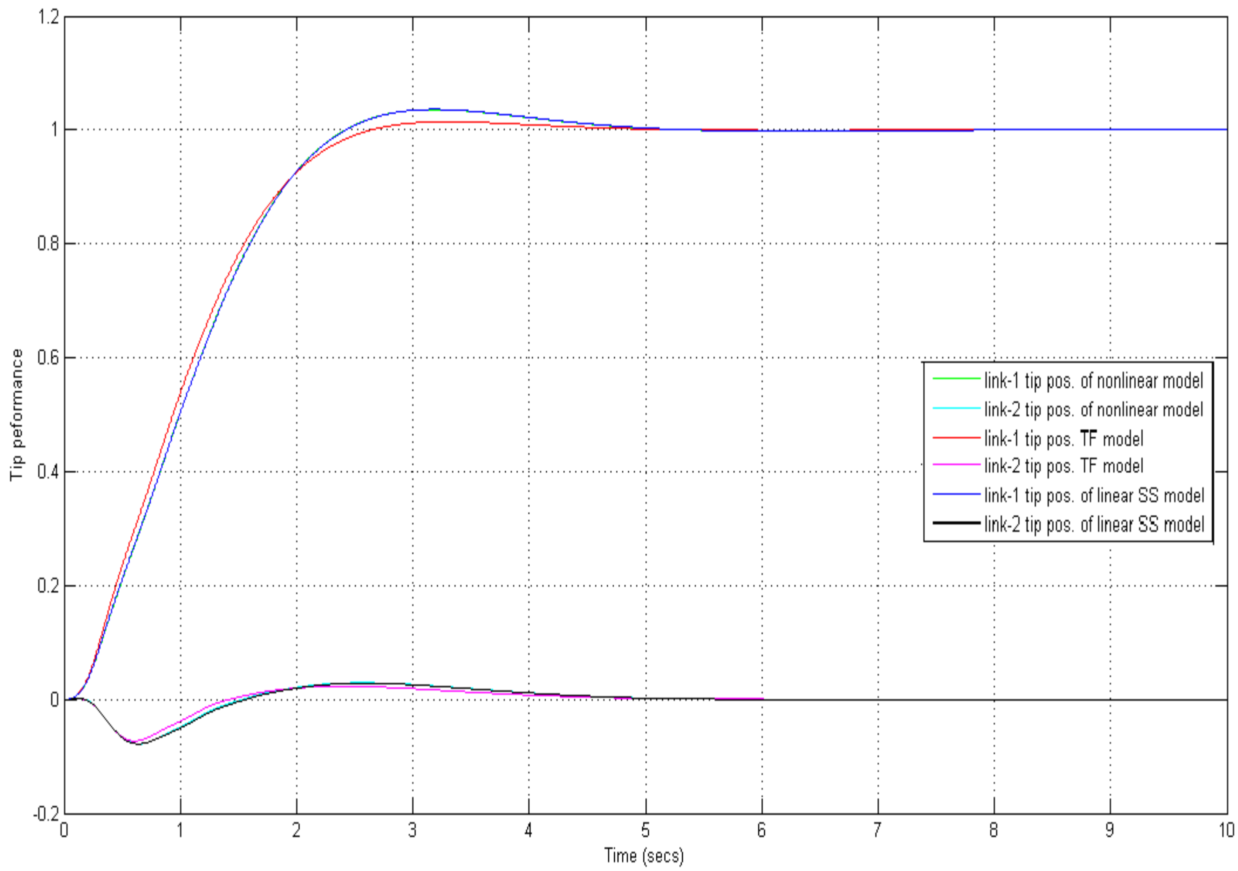


Fig.5.5: Tip performances of 3 different models using PD controller gains $K_p = 2.7$ and $K_d = 2.7$ with reference inputs to link-1 tip position as 1 and link-2 tip position as 0 for $M_P = 0.1$ kg.

The PD controller was initially designed for transfer function model by sensitivity and robust stability study. Then it was implemented on the nonlinear model. And as seen from the Fig.5.4, Fig.5.5 for different combination of tip position inputs this also gives the same output as that of the linear model.

Furthermore for robustness study the tip positions of both the links are plotted for a different value of the tip mass. The following figure presents the tip performances for $M_P = 60$ grams or 0.06 kg.

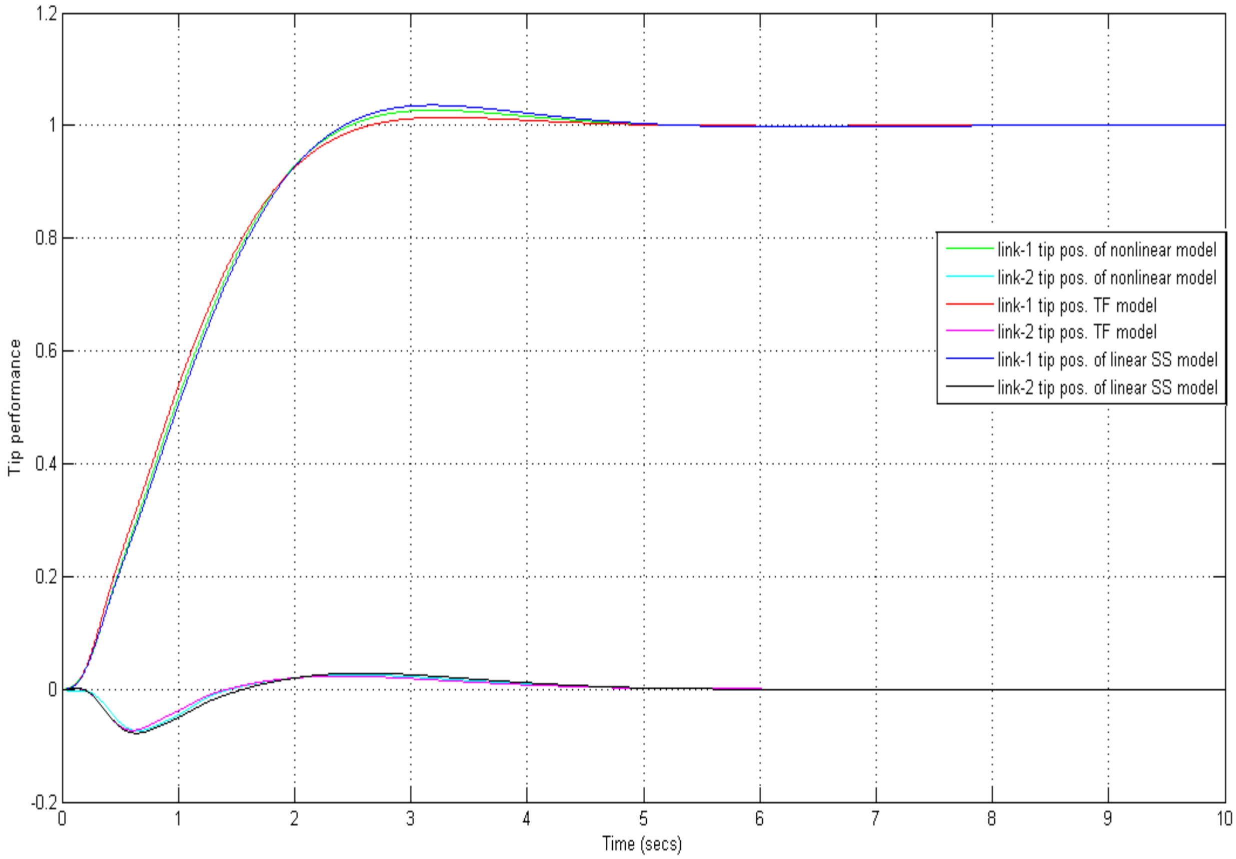


Fig.5.6: Tip performances of 3 different models using PD controller gains K_p = 2.7 and K_d = 2.7 with reference inputs to link-1 tip position as 1 and link-2 tip position as 0 for M_P = 0.06kg

From Fig.5.6 it is observed that the responses are almost similar to that of Fig.5.5, thus ensuring the robust stability for the system.

CHAPTER - 6

CONCLUSION

Conclusion

Modeling is the essence of interpreting a physical system in mathematical form which further facilitates in system analysis and controller design. The physical model of Two Link Flexible Manipulator (TLFM) has been analyzed and kinematic and Lagrangian modeling is done for the system to develop a dynamic model. The mathematical model obtained thus is found to be a nonlinear one. It is also found to be highly unstable and complex in nature. Using the system parameters the response of the nonlinear model is verified. From this model equivalent state space model is framed and then it is linearized about the equilibrium points. The resulting model is the linear model which responded well about the operating points and its neighborhood. Next a controller is designed for better tip performance and robust stability of the TLFM. For tip position control a simple and easily available PID controller design is proposed. But it is observed that the plant already has two poles at origin, so the objective becomes to design a PD controller rather than PID controller. Then for obtaining the gains of the controller, root locus loop shaping technique has been adopted. Robustness and stability criterion for the plant with PD controller are verified and positive inference is drawn. Finally this controller is employed to the nonlinear model and tip responses and robustness is analyzed.

References

[1]. A. De Luca and B. Siciliano, "Closed-Form Dynamic Model of Planar Multilink Lightweight Robots", *IEEE Trans. Syst., Man, Cybern.*, vol. 21, no. 4, pp. 826-839, 1991.

[2]. William A. Wolovich, *Automatic Control Systems Basic Analysis and Design*, Oxford University Press, 1995, pp. 219-223.

[3]. Kwanho You, *Adaptive Control*, In-Tech publication, 2009, pp. 87-112.

[4]. L. Meirovitch, *Analytical Methods in Vibrations*. New York: Macmillan, 1967.

[5]. R. H. Cannon, Jr. and E. Schmitz, "Initial experiments on the end-point control of a flexible one-link robot," *Int. J. Robotics Res.*, vol. 3, no.3, pp. 62-75, 1984.

[6]. G. G. Hastings and W. J. Book,"A linear dynamic model for flexible robotic manipulators," *IEEE Contr. Syst. Mag.*, vol. 7, no. 1, pp. 61-64, 1987.

[7]. I.J.Nagrath & M.Gopal, *Control Systems Engineering*, New Age Publisher, 2007.

[8]. M.O. Tokhi and A.K.M. Azad, *Flexible Robot Manipulators modeling, simulation and control*, IET, London, UK, 2008.